An Empirical Assessment of the Q-Factor Model: Evidence from the Karachi Stock Exchange

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Abstract

This paper tests the validity of the q-factor model on stocks listed on the Karachi Stock Exchange in Pakistan. The q-factor model is an investment-based factor model that explains stock returns based on market, profitability, investment and size factors and it tends to outperform the traditional CAPM, the Fama and French (1993) three-factor model and Carhart (1997) four-factor model, with some exceptions. While the model has been tested using data from stock markets in developed countries, the dynamics of emerging stock markets are significantly different, warranting a reapplication of the model to average stock returns in a developing market. We use data from the Karachi Stock Exchange to test the model in an emerging market context. The results show that, as firms increase their investment, their stock returns decline. Hence, a firm's investment is conditional on a given level of profitability. The size effect is strongly significant for small firms, but absent for large firms. Finally, the study identifies new factors that give a better understanding of returns in the context of an emerging economy such as Pakistan.

Keywords: Asset pricing, q-factor model, Karachi Stock Exchange, stock return.

JEL classification: G11, G12.

1. Introduction

This study investigates which factors determine the returns on stocks listed on the Karachi Stock Exchange (KSE) by applying the q-factor model developed by Hou, Xue and Zhang (2012, 2015). The q-factor model is an investment-based factor model derived from Tobin's q theory, which explains several anomalies of average returns not explained by earlier asset pricing models. This study empirically tests the validity of the factors identified by the q-factor model – market, investment, profitability and size – in relation to stocks listed on the KSE.

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Hou et al. (2012, 2015) developed a q-factor model that explains several average-return anomalies, most significantly the momentum effect. The q-factor model explains the impact of a firm's investment behavior and profitability on expected average stock returns – factors not explained by the Fama and French (1993) three-factor model. The q-factor model derives the investment and profitability factors and their relation to expected returns from Tobin's q theory (Tobin, 1969), which states that the firm's investment decisions depend on the ratio of the market value of capital to the replacement cost of capital – termed the marginal q. Firms tend to invest more when the marginal q is high and less when it is low. Similarly, all else equal, a high cost of capital means low investment and a low cost of capital means high investment.

The model considers the combined effect of profitability and investment because the relationship between the firm's investment and stock returns is conditional on a certain level of profitability, as high-investment firms tend to have higher levels of profitability (Fama & French, 2006). Thus, for a given expected profitability for the firm, its expected returns will decrease with increasing investment; for a given level of investment, its expected returns will increase with increasing profitability. Similarly, the positive relationship between profitability and returns is conditional on a given level of investment, as profitability may also be associated with higher investment. Specifically, the q-factor model states that the expected returns of a portfolio are explained by its sensitivity to four factors: market excess returns, the difference between the returns on small and large capitalization stocks, the difference between the returns on low and high investment-to-asset (I/A) stocks and the difference between the returns on high and low return-on-equity (ROE) stocks.

The three-factor model does not explain high asset growth in stocks (see Fama & French, 1993). Acknowledging the new q-factor model presented by Hou et al. (2012), but using a different theoretical framework, Fama and French (2015) test a comprehensive five-factor model that incorporates the investment and profitability effects into their earlier three-factor model. They find that the five-factor model outperforms the latter. However, the value factor becomes redundant and the five-factor model fails to capture the low returns on small stocks that have invested more despite low profitability. Accordingly, they suggest dropping the value factor if the objective is to measure regression intercepts, but retaining all five factors if the portfolios possess size, value, profitability and investment premiums. Fama and French (2017) tested the five-factor model for a sample drawn from North America, Europe and Japan, which largely

explained average returns, but with wide variability among factors across regions. The investment and profitability factors were strongest for North America, but insignificant for Japan and the Asia-Pacific region, where the value factor is strong.

Using data from the New York Stock Exchange, Hou, Xue and Zhang (2017) compared the performance of the q-factor model, the capital asset pricing model (CAPM), the Fama–French three-factor model, Carhart's (1997) four-factor model, Pástor and Stambaugh's (2003) model and Fama and French's (2015) five-factor model in explaining hundreds of stock return anomalies. They found that the two closely related investment-based models, the q-factor and five-factor models, outperform the others in explaining the maximum number of anomalies as well as expected returns. They termed the five-factor model a noisy version of the q-factor model. The q-factor model also explained the momentum effect identified by Jegadeesh and Titman (1993) better than other models, including that presented by Carhart et al. (1996), which also contains a momentum factor. While the q-factor model best captured the effect of momentum and profitability, the five-factor model explained value-versus-growth anomalies better than other models.

The q-factor model is a recent addition to the literature on asset pricing and there have been very few attempts to validate the model empirically. This study empirically tests the validity of the q-factor model based on the portfolio construction methodology developed by Fama and French (1993, 1996). Our sample consists of 100 companies listed on the KSE over the period June 2004 to May 2014. The objective is to determine the significance of the four factors identified by the q-factor model, i.e., market, size, investment and profitability. The study also estimates the explanatory power of excess market returns, small market capitalization (MC), a low I/A ratio and a high ROE ratio in KSE stock returns. This could help stock market investors determine whether one or more of the factors identified by the q-factor model can be used as a criterion for investment and portfolio formation. It will also help researchers and financial analysts better understand stock movements.

The next section discusses the relevant literature. Section 3 explains the theoretical framework. Section 4 describes the study's methodology and Section 5 discusses its findings. The article concludes with a set of policy implications, recommendations and limitations.

2. Theoretical Framework

The q-factor model generates hypotheses based on Tobin's q theory (see Tobin, 1969; Cochrane, 1991), while Fama and French (2015) derived their investment-based factor model from the dividend discount model presented by Gordon and Shapiro (1956).

2.1. Tobin's Q Theory

Tobin's (1969) theory of investment connects the financial market with the goods and services market. It suggests that the rate of investment is based on the ratio of the market value of a firm's capital to its replacement cost. Tobin's Q, also called average Q, is expressed as

$$Q = \frac{Market\ value\ of\ firm\ capital}{Replacement\ cost\ of\ capital} \tag{1}$$

Tobin's q theory provides certain guidelines for investment. The market value of the firm's capital is represented by the stock price. The firm's capital investment decision depends on where Q is in relation to 1. The theory states that firms should invest more capital when Q > 1 (which will bring it down to 1) and disinvest their capital stock when Q < 1 (which will raise it to 1). This means that capital has more value within the firm when Q > 1 and outside the firm when Q < 1. In a state of equilibrium, Q = 1 and there is no need for capital investment or disinvestment. However, an increase or decrease in capital is not free of cost. Tobin's q theory assumes that adjustment costs (such as installation costs) are associated with investment.

2.2. Dividend Discount Model

Hou et al. (2015) use the q theory of investment to derive their q-factor model. The model can also be estimated by applying the dividend discount model, following Fama and French (2015), according to which the worth of a stock is equal to the sum of all future dividends discounted to their present value (Gordon & Shapiro, 1956; Gordon, 1962). The model can be represented as follows:

$$P_0 = \frac{\sum_{n=1}^{\infty} E(d_t)}{(1+ke)^t} \tag{2}$$

where P_0 is the share price at time 0, $E(d_t)$ is the expected dividend per share at time t and ke is the internal rate of return. The dividend at time t

can be expressed as the difference between total and retained earnings (the portion of earnings that is reinvested).

The present market value of the firm is represented as:

$$P_0 = \frac{\sum_{t=1}^{\infty} TE_t - RE_t}{(1+ke)^t} \tag{3}$$

where TE_t denotes total earnings at time t and RE_t denotes retained earnings at time t. Retained earnings can be further expressed as the difference between the book values of equity.

According to Miller and Modigliani (1961), the market value of a stock can be shown as follows:

$$P_0 = \frac{\sum_{t=1}^{\infty} \left(TE_t - (B_t - B_{t-1}) \right)}{(1+ke)^t} \tag{4}$$

Here, $(B_t - B_{t-1})$ is the change in the book value of equity. The equation implies that higher earnings TE_t, reflected by profitability, will lead to higher expected returns, while higher growth in equity $(B_t - B_{t-1})$, i.e., higher investment, will lead to lower expected returns. We draw on the q-theory of investment as discussed above for our theoretical model.

3. Research Methodology

The four factors included in the q-factor model are market, size, investment and profitability. The market factor is derived from the CAPM. MC is used as a proxy for size, the I/A ratio as a proxy for investment and the ROE as a proxy for profitability. The study employs stock portfolios instead of individual stocks. Blume (1970) suggests that the motivation for creating stock portfolios is to reduce idiosyncratic risk, as the errors of individual stocks will offset each other if they are grouped in a portfolio. The three factors are constructed using the standard methodology developed by Fama and French (1993, 1996).

3.1. Data Source and Sample

The data was obtained from companies' financial statements, the KSE data portal and the State Bank of Pakistan's website over the sample period. The population consisted of all the stocks listed on the KSE during 2004–14. The number of listed companies in 2004 was 701, which dropped to 600 by 2014 (Securities and Exchange Commission of Pakistan, 2014). Thus, any companies that had been delisted for any

reason were not included in the study. The population also excluded 11 nonfinancial company sectors whose earnings did not rely on capital investments, namely, banks, development finance institutions, microfinance banks, leasing companies, investment banks, mutual funds, *modarabas*, exchange companies, insurance companies, housing finance and venture capital (State Bank of Pakistan, 2015). Additionally, any stocks with negative book equity were omitted.

The sample, which represents about 25 percent of the population, consists of 100 stocks listed on the KSE for the period starting June 2004 and ending May 2014. According to Hair et al. (2010), the suggested ratio of observations to the number of predictors is 15 to 20. We have data from 100 companies and four predictors, giving us an observation-to-predictor ratio of 25 – well above the minimum requirement. The stocks were selected using simple random sampling. This was done by assigning a serial number to each firm and then randomly selecting 100 serial numbers. The nonfinancial sectors from which the data was collected are listed in Table 1.

Table 1: Sectors of nonfinancial companies listed on the KSE

Sector	Number of companies
Textiles	152
Sugar	31
Food	16
Chemicals, chemical products and pharmaceuticals	45
Manufacturing	32
Mineral products	8
Cement	20
Motor vehicles, trailers and auto parts	20
Fuel and energy	22
Information, communication and transport services	13
Coke and refined petroleum products	10
Paper, paperboard and products	9
Electrical machinery and apparatus	7
Other services activities	11

Source: State Bank of Pakistan (2015).

The unit of study is the portfolio (formed by combining a group of stocks). A portfolio comprises stocks with similar characteristics such as size, investment and profitability. The sample portfolios are dynamic and updated each year to maintain their specific characteristics. Thus, a stock

whose characteristics have changed over the year can jump from one portfolio to another.

3.2. Excess Monthly Returns

We use a one-month horizon for the tests by taking the closing price on the last day of each month. The monthly stock returns are calculated by applying the formula given in equation (5):

$$R_t = \frac{(P_t - P_{t-1})}{P_{t-1}} \tag{5}$$

where R_t is the stock return in the current month t, P_t is the closing price of stock i at the end of the current month t and P_{t-1} is the closing price of stock i at the end of the previous month $t-1$.

The benchmark KSE 100 index is taken as a proxy for the market portfolio. The monthly market returns are calculated by dividing the change in the KSE 100 over a month by its closing value for the previous month:

$$R_{mt} = \frac{(KSE_t - KSE_{t-1})}{KSE_{t-1}} \tag{6}$$

where R_{mt} is the return on the market portfolio in the current month t, KSE_t is the closing value of the KSE 100 at the end of the current month t and KSE_{t-1} is the closing value of the KSE 100 at the end of the previous month $t-1$.

The risk-free return is subtracted from the value-weighted return on the portfolio to obtain the excess return. The three-month T-bill rate is used as a proxy for the risk-free rate (see Harrington, 1987). Similarly, the excess market return is calculated by subtracting the risk-free return from the market return. The annualized three-month T-bill return is converted into a monthly return using the formula below:

$$Monthly\ return = (1 + annual\ return)^{\frac{1}{12}} - 1 \tag{7}$$

3.3. MC, ROE and I/A Ratio

The value of MC at the end of each year is obtained by multiplying the stock price by outstanding shares:

$$MC = share\ price \times outstanding\ shares$$

The current value of ROE is obtained by dividing net income by one-year-lagged book equity:

$$ROE_t = \frac{net\ income}{book\ equity_{t-1}} \tag{8}$$

The I/A ratio is calculated as the annual change in gross property, plant and equipment plus the annual change in inventory divided by the lagged book value of assets (see Chen, Novy-Marx & Zhang, 2011). The change in property, plant and equipment is taken as a standard measure of firm-level investment (Eberly, Rebelo & Vincent, 2008). The change in inventory captures investment in short-lived assets during an operating cycle:

$$\frac{I}{A} = \frac{(PPE_t - PPE_{t-1}) + (inv_t - inv_{t-1})}{Total\ assets_{t-1}} \tag{9}$$

3.4. Portfolio Formation

The percentile technique is used to sort the 100 stocks by size, investment and profitability. The portfolios are constructed by a triple two-by-three-by-three sorting of MC, the I/A ratio and ROE. For the size factor, the stocks are split into two groups by applying a breakpoint at the 50th percentile of the ranked values of MC. For the investment factor, the stocks are split into three groups, using breakpoints at the 30th and 70th percentiles of the ranked values of the I/A ratio. For the profitability factor, the stocks are split into three groups, using breakpoints at the 30th and 70th percentiles of the ranked values of ROE. Two MC, three I/A ratio and three ROE groups intersect to create 18 portfolios. Each portfolio is created by combining stocks whose MC, I/A ratio and ROE intersect with each other. These portfolios are presented in Table 2.

Table 2: Portfolios ranked by MC, I/A ratio and ROE

I/A	ROE			MC
	1st tercile	2nd tercile	3rd tercile	
1st tercile	P1	P3	P5	1st median
	P2	P4	P6	2nd median
2nd tercile	P7	P9	P11	1st median
	P8	P10	P12	2nd median
3rd tercile	P13	P15	P17	1st median
	P14	P16	P18	2nd median

Source: Authors' estimates.

Since it is possible for the sampled companies to change rankings due to changes in size, investment and profitability, we revise the ranking and sorting of stocks for every year in June. This allows stocks to move freely from one portfolio to another at the end of each year. The stocks are required to match the characteristics of the corresponding I/A ratio and ROE tercile or MC median only for the current year.

3.5. Value-Weighted Portfolio Returns

The weighted average monthly returns of each portfolio are calculated by assigning weights based on the MC of their constituent stocks. The large-capitalization stocks in each portfolio have greater weight than the smaller ones and thus contribute more to total portfolio returns.

3.6. Factor Construction

The stocks in the first ROE tercile (P1, P2, P7, P8, P13, P14) are low-profitability stocks and those in the third ROE tercile (P5, P6, P11, P12, P17, P18) are high-profitability stocks. The stocks in the first I/A tercile (P1, P2, P3, P4, P5, P6) are low-investment stocks and those in the third I/A tercile (P13, P14, P15, P16, P17, P18) are high-investment stocks. The stocks in the first MC median (P1, P3, P5, P7, P9, P11, P13, P15, P17) are small stocks and those in the second MC median (P2, P4, P6, P8, P10, P12, P14, P16, P18) are large stocks.

The size factor in the q-factor model is represented by MCSMB. MCSMB is constructed by subtracting the average returns of all nine small-capitalization portfolios from the average returns of all nine large-capitalization portfolios for each month. The subtraction represents taking a long position on small portfolios and a short position on large portfolios. The investment factor is represented by I/ALMH. I/ALMH is formulated by subtracting the average returns of all six high-I/A ratio portfolios from the average returns of all six low-I/A ratio portfolios for each month. Here, subtraction represents taking a long position on low-investment portfolios and a short position on high-investment portfolios. The profitability factor is represented by ROEHML. ROEHML is formulated by subtracting the average returns of all six low-ROE portfolios from the average returns of all six low-ROE portfolios for each month. Here, subtraction represents taking a long position on high-profitability portfolios and a short position on low-profitability portfolios.

3.7. Empirical Model

The q-factor model has the following multivariate linear expression:

$$E(R_p) - R_f = \beta_o + \beta_1(R_m - R_f) + \beta_2 MC_{SMB} + \beta_3 I/A_{LMH} + \beta_4 ROE_{HML} + \epsilon \qquad (10)$$

where $E(R_p)$ = expected return of portfolio p, R_f = monthly risk-free rate of return, $E(R_p) - R_f$ = monthly excess return on portfolio, $(R_m - R_f)$ = monthly excess market return, MC_{SMB} = long position on small portfolios and short position on large portfolios, I/A_{LMH} = long position on low-investment portfolios and short position on high-investment portfolios, and ROE_{HML} = long position on high-profitability portfolios and short position on low-profitability portfolios. β_1, β_2, β_3 and β_4 are the regression coefficients of the independent variables, β_0 is the intercept and ϵ is the error term.

The regression is applied to all 18 portfolios. The monthly excess portfolio returns are taken as the dependent variable. $(R_m - R_f)$, MC_{SMB}, I/A_{LMH} and ROE_{HML} are the independent variables. The 18 portfolios are also tested using the CAPM and the results compared with those for the q-factor model.

3.8. Hypotheses

The hypotheses we test are as follows:

- H$_1$: The excess returns of the portfolios are positively related to excess market returns.

- H$_2$: The excess returns of small-MC portfolios are positively related to MCSMB.

- H$_3$: The excess returns of low-I/A portfolios are positively related to I/ALMH.

- H$_4$: The excess returns of high-ROE portfolios are positively related to ROEHML.

4. Empirical Results

To test the validity of the q-factor model using data from the KSE 100, we estimate equation (10) for each of the 18 portfolios. This involves multiple regression analysis with robust standard errors, given that heteroskedasticity is observed in eight of the portfolios. The descriptive

statistics in Table 3 show that the mean values of the average returns vary from 1.004 to 1.040 with a standard deviation of 0.07–0.136. The minimum return is 0.57 and the maximum is 1.95.

Table 3: Descriptive statistics

Portfolio	Obs.	Mean	SD	Min	Max
P1	120	1.031038	0.1056943	0.83888	1.51049
P2	120	1.019340	0.0875300	0.79465	1.22562
P3	120	1.025833	0.0963303	0.80818	1.38102
P4	120	1.020513	0.0871659	0.67345	1.28651
P5	120	1.015764	0.0894201	0.69221	1.26382
P6	120	1.014576	0.0896182	0.57560	1.42166
P7	120	1.005441	0.0799410	0.75066	1.24733
P8	120	1.011772	0.0902566	0.61363	1.25116
P9	120	1.017096	0.0731724	0.81726	1.21686
P10	120	1.014072	0.0776979	0.82504	1.32364
P11	120	1.032450	0.1124963	0.82307	1.78466
P12	120	1.022794	0.0744963	0.85264	1.27880
P13	120	1.019339	0.1118930	0.71383	1.49122
P14	120	1.004073	0.0867751	0.75049	1.25894
P15	120	1.021825	0.0860618	0.81543	1.32749
P16	120	1.017632	0.0856179	0.69147	1.24887
P17	120	1.019526	0.1000590	0.79966	1.42179
P18	120	1.041620	0.1359149	0.66342	1.95311

Source: Authors' estimates.

The results of these estimations are given in Tables 4, 5 and 6. The market factor (β_1) is positive and significant at 1 percent for all portfolios except P5. For all nine large portfolios, the market factor has the highest beta of all four factors being tested in the q-factor model. However, the average size factor (β_4) of the nine small portfolios is stronger than their average market factor. The average β_1 for the 18 portfolios is 0.8769. The highest β_1 is for portfolio P13 and the lowest β_1 for P9. There is no marked variation in the strength of market betas across the portfolios.

Table 4: Q-factor model estimations P1 to P6

	P1	P2	P3	P4	P5	P6
β_1	0.8798***	0.8934***	0.7039***	0.9881***	0.9652	0.9511***
	(0.102)	(0.078)	(0.097)	(0.07)	(0.07)	(0.052)
β_2	0.1433	0.2161*	0.2606*	0.0165	0.3759***	0.2295***
	(0.253)	(0.114)	(0.142)	(0.096)	(0.103)	(0.076)
β_3	-0.2158	-0.356***	-0.0811	-0.2029**	0.2130**	0.3382***
	(0.332)	(0.102)	(0.126)	(0.095)	(0.091)	(0.068)
β_4	1.3732***	0.2378***	1.2639***	0.1416	0.5266***	-0.2058**
	(0.307)	(0.127)	(0.158)	(0.111)	(0.114)	(0.085)
β_5	0.0137***	0.0059	0.00904	0.00511	-0.00442	-0.0048
	(0.006)	(0.005)	(0.007)	(0.005)	(0.005)	(0.004)
F-value	33.43	39.34	24.12	55.06	59.39	131.03
(4, 115)	p = 0.000	p = 0.000	p = 0.000	p = 0.000	p = 0.000	p = 0.000
R^2	0.5056	0.5778	0.4562	0.6563	0.6738	0.8201
n	120	120	120	120	120	120

Note: Standard errors are given in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
Source: Authors' estimates.

The investment factor (β_2) is significant for 13 out of 18 portfolios. The five portfolios with insignificant investment factors include two low-investment (P1, P4) and two medium-investment (P10, P12) portfolios. The average β_2 for the six low-investment portfolios is 0.207, while the average β_2 for the six high-investment portfolios is –0.793. The investment factor has the strongest positive value for the low-investment portfolio P5 and the strongest negative value for the high-investment portfolio P18. The results show that four of the six low-investment portfolios have positive coefficients and all six high-investment portfolios (P13, P14, P15, P16, P17, P18) have negative and significant (at 1 percent) coefficients. Thus, as the firm increases its investment, its expected stock returns fall.

The profitability factor (β_3) is significant for 13 out of 18 portfolios. The four portfolios for which the profitability factor is insignificant include two low-profitability (P1, P3) and two high-profitability (P15, P16) portfolios. The profitability factor is significant for all portfolios with medium profitability. The average β_3 for the six low-profitability portfolios is –0.0507 and for the six high-profitability portfolios is –0.0507. The coefficient is smallest for portfolio P13 and largest for the high-profitability portfolio P18. These findings are consistent with the model's predictions: low-profitability portfolios have a negative β_3 and high-profitability

portfolios have a positive β_3, with some exceptions. Thus, as the firm's profitability increases, its expected stock returns also increase.

Table 5: Q-factor model estimations P7 to P12

	P8	P9	P10	P11	P12
β_1	0.9451***	0.6996***	0.7769***	0.9264***	0.7388***
	(0.118)	(0.073)	(0.077)	(0.097)	(0.062)
β_2	-0.4041***	-0.253***	0.1361	-0.3741*	0.0515
	(0.132)	(0.107)	(0.107)	(0.209)	(0.09)
β_3	-0.6064***	-0.2152**	-0.2752**	0.6264**	0.2489***
	(0.12)	(0.095)	(0.111)	(0.279)	(0.08)
β_4	-0.0673	0.6360***	-0.0561	1.5500***	-0.0432
	(0.133)	(0.118)	(0.164)	(0.283)	(0.1)
β_5	0.0012	0.00338	0.00169	0.00667	0.00591
	(0.006)	(0.005)	(0.005)	(0.006)	(0.004)
F-value	27.97	25.71	29.86	24.44	49.57
(4, 115)	p = 0.000	p = 0.000	p = 0.000	p = 0.000	p = 0.000
R^2	0.625	0.4721	0.5899	0.6225	0.6329
n	120	120	120	120	120

Note: Standard errors are given in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
Source: Authors' estimates.

Table 6: Q-factor model estimations P13 to P18

	P13	P14	P15	P16	P17	P18
β_1	1.0098***	0.8412***	0.857***	0.8716***	0.976***	0.8258***
	(0.102)	(0.07)	(0.069)	(0.076)	(0.077)	(0.135)
β_2	-1.008***	-0.757***	-0.607***	-0.309***	-0.726***	-1.351***
	(0.18)	(0.083)	(0.101)	(0.007)	(0.113)	(0.269)
β_3	-0.859***	-0.628***	-0.0778	-0.1196	0.4585***	0.9224***
	(0.158)	(0.07)	(0.09)	(0.1)	(0.101)	(0.317)
β_4	1.1099***	-0.0757	1.076***	-0.1465	0.9995***	0.3738
	(0.17)	(0.107)	(0.112)	(0.124)	(0.125)	(0.32)
β_5	0.008	-0.0051	0.0045	0.0035	-0.0036	0.0174***
	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)	(0.008)
F-value	32.05	52.92	54.46	38.82	61.85	18.09
(4, 115)	p = 0.000	p = 0.000	p = 0.000	p = 0.000	p = 0.000	p = 0.000
R^2	0.6819	0.6157	0.6545	0.5745	0.6827	0.5742
n	120	120	120	120	120	120

Note: Standard errors are given in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
Source: Authors' estimates.

The size factor is significant for 10 out of 18 portfolios. The size factor is significant for four of the five small portfolios and tends to be stronger than their market factors. The average β_4 for the small portfolios is 1.017 while the average β_4 for the nine large portfolios is 0.017. The largest size coefficient is for the small portfolio P11 and the smallest size coefficient is for the large portfolio P6. These findings confirm the model's predictions: small portfolios have a strong, positive β_4 and large portfolios have an insignificant or negative β_4. Thus, as the firm's size increases, its expected stock returns decrease.

In a perfect asset-pricing model explaining excess returns above the risk-free rate, the value of the intercept must be close to 0 (Black, Jensen & Scholes, 1972). A zero-intercept is based on the risk-return relationship according to which there should be no return on taking no risk. A non-zero intercept indicates the model's failure to explain excess returns. The same rationale is extended to the q-factor model such that the intercept value is expected to be 0. The results show that the intercept values for all the portfolios are close to 0. This implies that the model is specified correctly (see Hou et al., 2017) and that it explains excess returns without needing additional variables.

The F-test for all 18 regressions is significant, with a p-value of 0.000. The average R-squared for all portfolios is 0.61. The highest R-squared is 0.82 for portfolio P6 and the smallest is 0.45 for portfolio P3. The R-squared values lie in a similar range for all groups of portfolios ranked by investment, size and profitability. The data used for these estimations is also used to estimate a CAPM, the results of which are significant for all 18 portfolios (Table 7). The F-test for each regression is significant, with a p-value of 0.000. The average R-squared for all portfolios is 0.38. Overall, the q-factor model has greater explanatory power than the CAPM.

Table 7: CAPM regression analysis

Portfolio	Intercept	p-value	β	p-value	R2	F test	p-value
P1	0.01773	0.048	0.5912	0.000	0.172	24.42	0.000
P2	0.00384	0.508	0.8306	0.000	0.490	113.40	0.000
P3	0.01368	0.102	0.4658	0.000	0.128	17.34	0.000
P4	0.00402	0.411	0.9388	0.000	0.633	203.80	0.000
P5	-0.00058	0.914	0.9235	0.000	0.580	162.80	0.000
P6	-0.00298	0.467	1.0564	0.000	0.757	367.70	0.000
P7	-0.00821	0.175	0.6307	0.000	0.339	60.56	0.000
P8	-0.00386	0.522	0.8466	0.000	0.480	108.90	0.000
P9	0.00447	0.441	0.5168	0.000	0.271	43.82	0.000
P10	-0.00090	0.853	0.7740	0.000	0.541	139.20	0.000
P11	0.01875	0.049	0.6341	0.000	0.174	24.91	0.000
P12	0.00775	0.078	0.7807	0.000	0.600	176.90	0.000
P13	0.00620	0.518	0.5730	0.000	0.144	19.77	0.000
P14	-0.01023	0.115	0.7016	0.000	0.356	65.17	0.000
P15	0.00882	0.209	0.5583	0.000	0.230	35.21	0.000
P16	0.00193	0.719	0.8532	0.000	0.542	139.60	0.000
P17	0.00487	0.532	0.7382	0.000	0.296	49.71	0.000
P18	0.02736	0.020	0.6950	0.000	0.143	19.70	0.000

Source: Authors' estimates.

Post-estimation, we test for multicollinearity using the variance inflation factor test. No multicollinearity is observed in the model (see Table A1 in the Appendix). The Breusch–Godfrey LM test results show that there is no autocorrelation at the first lag. Campbell et al. (2001) divide stock volatility into three components – market, industry and idiosyncratic, all of which exhibit time variation. The Breusch–Pagan test is carried out to test for heteroskedasticity, which emerges in eight out of 18 portfolios (see Table A2 in the Appendix). To remove the impact of heteroskedasticity on the estimators, we carry out the regressions with robust standard errors.

5. Discussion

This empirical study applies the q-factor model to a sample of stocks listed on the KSE by analyzing data from 100 companies for the period June 2004 to May 2014. The analysis involves running regressions on 18 portfolios with distinct characteristics. The average R-squared for all portfolios using the q-factor model is 0.61, while that for all portfolios using the CAPM is 0.38. This implies that the q-factor model vastly outperforms the CAPM. Since the intercept of the q-factor model is close to 0, we can assume the model is accurately specified.

The q-factor model consists of four factors: market, investment, profitability and size. The effect of the market factor on expected returns is positive and significant for all portfolios except one (P5). The effect of the investment factor on expected returns is significant for 13 out of 18 portfolios. It has a negative and significant effect for all six high-investment portfolios. The results show that, when the firm's level of investment is low, the effect of investment on expected market returns is insignificant or positive. However, as the firm increases its investment, expected returns decline.

The ideal portfolio according to the q-factor model is P6, which features small, low-investment, high-profitability stocks. All its factors are significant and consistent with the model's predictions (positive investment, profitability and size factors). Conversely, the worst portfolio according to the q-factor model is P14, which comprises large, high-investment, low-profitability stocks. Its investment and profitability factors have negative coefficients, while the size factor is insignificant. Overall, the q-factor model accurately forecasts the returns on stock portfolios with varying characteristics, while these results confirm the negative-investment-and-expected-return relationship.

The profitability factor is significant for 13 out of 18 portfolios. The low-profitability portfolios have negative coefficients and the high-profitability portfolios have positive coefficients. This means that low-profitability portfolios have a negative effect on expected returns, whereas an increase in profitability has a positive effect on expected returns for firms with high profitability. This implies there is a positive relationship between profitability and expected returns – a finding consistent with Fama and French (2015) and Hou et al. (2015, 2017). The size factor is significant for 10 out of 18 portfolios. It is insignificant for seven out of nine large portfolios, but positive for all nine small portfolios – in which size is the strongest of all four factors. The market factor is positive and significant at 1 percent for all portfolios except P5. For all nine large portfolios, the market factor has the highest betas of all four factors being tested in the q-factor model.

Studies using q-factor or similar models have been carried out for different countries/stock markets. Using weighted portfolios, Chen et al. (2011) investigate which factors explain the variations in a cross-section of expected returns on stocks listed on the NYSE, AMEX and NASDAQ. They find that investment and profitability explain most of the anomalies observed, including momentum, accruals, net stock issues and asset growth. They term this model the 'alternative' three-factor model and

show that it yields significantly better results than the Fama–French three-factor model for stocks in the US.

Ammann, Odonia and Oesch (2012) evaluate the performance of an investment-based factor model by employing the I/A ratio and ROE for a sample of European stock markets (Austria, Belgium, Finland, France, Germany, Italy, Ireland, the Netherlands, Portugal and Spain) over 1990–2006. They find that the investment-based model performs better than the CAPM or Fama–French three-factor model in explaining asset return anomalies such as asset growth, short-term prior returns, net stock issues, total accruals and value effects. Fan and Yu (2013) investigate the momentum anomaly otherwise not explainable by the CAPM and Fama–French three-factor model. They use the investment-based alternative three-factor model and find that it explains the momentum anomaly in 12 out of 13 G-12 country stock markets and yields significantly lower intercept values.

Fama and French (2015) add the investment and profitability factors derived from the q theory of investment to their earlier three-factor model to form a comprehensive five-factor model directed at capturing the impact of size, value, profitability and investment. They find that it outperforms the earlier model. Their results also indicate that small high-investment stocks have lower returns than high-investment, low-profitability stocks. However, the value factor becomes redundant in the presence of the other four factors, especially investment and profitability.

Finally, Walkshäusl and Lobe (2014) test the q theory-based model, which employs investment and profitability, and the Fama–French three-factor model for a global portfolio of 40 non-US markets in emerging and developed countries. They conclude that the q theory-based model outperforms the three-factor model in capturing the momentum anomaly, but has less explanatory power in relation to average returns. This could mean that the investment-based model is sample-specific.

6. Conclusion

This study contributes to the literature by validating the factors identified in the q-factor model as predictors of the expected returns on investments in the KSE. This implies that the four factors taken up in the q-factor model are useful predictors of average returns not only in developed markets, but also in developing markets. The model adds investment and profitability as predictors of expected market returns in factor-based asset pricing.

Our results are largely in accordance with the q theory and other findings relevant to US and other markets, where all the factors are found to be significant (see Hou et al., 2015, 2017). In a study on the Vietnamese stock market, however, Nguyen, Ulku and Zhang (2015) show that profitability and investment are important determinants of asset returns, along with size and value. The Vietnamese stock market is distinct from other stock markets in that the state owns a large volume of stocks.

Of the two new factors identified in the q-factor model, profitability (measured by ROE) has been traditionally used in fundamental analysis. However, our results show that investment, represented by the I/A ratio, can also be used as a tool of fundamental analysis for individual stocks. Further, investors can trade against the investment and profitability factors to increase their returns. Finally, the q-factor model has better explanatory power than the traditional CAPM and can be used to explain various anomalies, allowing better portfolio valuation.

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Appendix

Table A1: Vector inflation factors of variables

Variable	VIF	1/VIF
Market factor	1.17	0.855147
Size factor	1.16	0.865533
Profitability factor	1.08	0.928310
Investment factor	1.08	0.928425
Mean VIF	1.12	

Source: Authors' estimates.

Table A2: Breusch–Pagan/Cook–Weisberg test for heteroskedasticity

Portfolio	chi2(1)	Prob. > chi2	Portfolio	chi2(1)	Prob. > chi2
P1	8.77	0.0031*	P10	6.44	0.0111*
P2	0.67	0.4134	P11	66.96	0.0000*
P3	3.01	0.0829	P12	1.44	0.2308
P4	17.44	0.0000*	P13	7.06	0.0079*
P5	1.36	0.2441	P14	5.94	0.0148*
P6	1.56	0.2121	P15	2.95	0.0859
P7	0.04	0.8326	P16	2.04	0.1535
P8	12.28	0.0005*	P17	1.67	0.1959
P9	0.80	0.3726	P18	51.32	0.0000*

Note: * Heteroscedasticity is present.
Source: Authors' estimates.